European Economic Review 56 (2012) 727–746

Contents lists available at SciVerse ScienceDirect

European Economic Review

journal homepage: www.elsevier.com/locate/eer

Does intellectual monopoly stimulate or stifle innovation?

Angus C. Chu [a,b,c,1], Guido Cozzi [a,*], Silvia Galli [d,2]

[a] Durham Business School, Durham University, Durham, UK
[b] School of Economics, Shanghai University of Finance and Economics, Shanghai, China
[c] Institute of Economics, Academia Sinica, Taipei, Taiwan
[d] Hull University Business School, University of Hull, Hull, UK

ARTICLE INFO

Article history:
Received 20 May 2011
Accepted 21 January 2012
Available online 5 February 2012

JEL classification:
O31
O34
O40

Keywords:
Economic growth
Innovation
Intellectual property rights

ABSTRACT

This study develops an R&D-based growth model with vertical and horizontal innovation to shed some light on the current debate on whether patent protection stimulates or stifles innovation. We analyze the effects of patent protection in the form of blocking patents. We show that patent protection changes the direction of innovation by having asymmetric effects on vertical innovation (i.e., quality improvement) and horizontal innovation (i.e., variety expansion). Calibrating the model and simulating transition dynamics, we find that strengthening the effect of blocking patents stifles vertical innovation and decreases economic growth but increases social welfare due to an increase in horizontal innovation. In light of this finding, we argue that in order to properly analyze the growth and welfare implications of patents, it is important to consider their often neglected compositional effects on vertical and horizontal innovation.

1. Introduction

Since the early 1980s, the patent system in the US has undergone substantial changes.[3] As a result of this patent reform, the strength of patent protection in the US has increased. For example, Park (2008) provides an index of patent rights on a scale of 0–5 (a larger number implies stronger protection) and shows that the strength of patent rights in the US increases from 3.8 in 1975 to 4.9 in 2005.[4] In other words, patentholders can now better to protect their inventions against imitation as well as *subsequent innovation*. When a patent protects an invention against subsequent innovation, a blocking patent arises. A classic example of blocking patents is James Watt's patent on his steam engine. Boldrin et al. (2008) argue that "[b]y patenting the separate condenser Boulton and Watt, from 1769 to 1800, had almost absolute control on the development of the steam engine. They were able to use the power of their patent and the legal system to frustrate the efforts of engineers such as Jonathan Hornblower to further improve the fuel efficiency of the steam engine." As for the current patent system, economists have become even more concerned about the innovation–stifling effect of blocking patents. For example, Shapiro (2001) argues that "[w]ith cumulative innovation and multiple blocking patents, stronger

* Corresponding author. Tel.: +44 1913 345146; fax: +44 1913 345201.
E-mail addresses: anguscccc@gmail.com (A.C. Chu), guido.cozzi@durham.ac.uk (G. Cozzi), s.galli@hull.ac.uk (S. Galli).
[1] Tel.: +44 1913 346374; fax: +44 1913 345201.
[2] Tel.: +44 1482 463239; fax: +44 1482 463484.
[3] See Gallini (2002), Jaffe (2000) and Jaffe and Lerner (2004) for a detailed discussion on these changes in patent policy.
[4] The index in Park (2008) is an updated version of the index in Ginarte and Park (1997), who examine five categories of patent rights and assign a score from zero to one to each category. These five categories are patent duration, coverage, enforcement mechanisms, restrictions on patent scope, and membership in international treaties.

0014-2921/$ - see front matter
doi:10.1016/j.euroecorev.2012.01.007

patent rights can have the perverse effect of stifling, not encouraging, innovation.'' In this study, we provide a growth-theoretic analysis on the effects of patent protection in the form of blocking patents.

In an environment with cumulative or sequential innovation, blocking patents give rise to overlapping patent rights across sequential innovators and lead to contrasting effects on R&D. On the one hand, the traditional view suggests that stronger patent rights improve the protection for existing inventions and increase their value to the patentholders. On the other hand, the recent argument against patent protection suggests that stronger patent rights stifle innovation by giving too much power to existing patentholders, who use this power to extract surplus from subsequent innovators rather than providing more innovation.[5] In this study, we develop a simple growth model to shed some light on this current debate on whether patents stimulate or stifle innovation. We argue that the two seemingly contradictory views of patents are in fact two sides of the same coin. In other words, strengthening existing patentholders' protection against future innovations inevitably decreases subsequent innovators' incentives for R&D and leads to contrasting effects on vertical innovation (i.e., quality improvement within an industry) and horizontal innovation (i.e., variety expansion that gives rise to new industries). In light of this finding, we argue that in order to properly analyze the growth and welfare implications of patents, it is important to consider their often neglected compositional effects on vertical and horizontal innovation.

To analyze the asymmetric effects of patent protection on vertical and horizontal innovation, this study develops an R&D-based growth model that features both quality improvement and variety expansion. Within this framework, we derive the growth and welfare effects of patent protection in the form of blocking patents. A strengthening of blocking patents refers to the case in which a new innovator (e.g., Jonathan Hornblower) has to transfer a larger share of his profit to the previous innovator (e.g., James Watt). We find that there is a tension between maximizing the incentives for vertical innovation and that of horizontal innovation. On the one hand, maximizing the incentives for vertical innovation requires a profit-division rule that allows the new innovator to keep all the profit. On the other hand, maximizing the incentives for horizontal innovation requires a profit-division rule that assigns as much profit to the previous innovator as possible. As a result of these asymmetric effects on vertical and horizontal innovation, strengthening the effect of blocking patents stimulates variety expansion but stifles quality improvement affecting the direction of innovation. This theoretical result is consistent with the empirical finding in Moser (2005), who provides an empirical analysis on how patent protection affects the direction of innovation and finds that the presence of patent laws in a country causes the inventions to be more diversified and directed to a broader set of industries than inventions in countries without patent laws.

Furthermore, strengthening the effect of blocking patents has an additional effect through horizontal innovation on social welfare by increasing the number of varieties, so that there also exists a welfare-maximizing profit-division rule that is generally different from the growth-maximizing rule. Calibrating the model and simulating transition dynamics, we find that an increase in the effect of blocking patents stifles vertical innovation and decreases the overall growth rate despite the increase in horizontal innovation. This finding is consistent with the recent concerns on the innovation–stifling effects of stronger patent rights. However, we also find that social welfare increases despite the lower growth rate suggesting that a proper welfare analysis should investigate beyond the effects of patent protection on innovation and economic growth.

Nordhaus (1969) is the seminal study on the optimal design of patent protection, and he shows that the optimal patent length should balance between the social benefit of innovation and the social cost of monopolistic distortion. Eswaran and Gallini (1996) analyze the interesting interaction between product and process innovations and consider patent breadth as a policy tool that can be used to redirect technological change toward a socially efficient mix of innovations. Scotchmer (2004) provides a comprehensive review on the subsequent development in this patent-design literature that is mostly based on partial-equilibrium models. In this literature, an interesting and important policy lever is forward patent protection (i.e., leading patent breadth) that gives rise to the division of profit between sequential innovators.[6] A recent study by Segal and Whinston (2007) analyzes a general antitrust policy lever that has a similar effect as the division of profit between entrants and incumbents. They show that in an infinite-horizon model with leapfrogging, protecting an entrant at the expense of an incumbent has a frontloading effect that potentially increases innovation. However, they also note that their result does not apply to the first firm of a quality ladder because it does not have to share its profit with any incumbent but has the rights to share the next entrant's profit. In the present study, we formalize Segal and Whinston's interesting insight in a dynamic general-equilibrium model and match the model to the US data in order to provide a quantitative analysis on the division of profit between sequential innovators.

O'Donoghue and Zweimuller (2004) merge the patent-design literature and the R&D-based growth literature by incorporating leading breadth into a quality-ladder growth model with overlapping patent rights across sequential innovators. In their model, for a given rate of innovation, strengthening the effect of blocking patents by reducing the share of profit assigned to the current innovator (i.e., the entrant of a quality ladder) while holding leading breadth constant would decrease the incentives for innovation. Intuitively, along the quality ladder, every innovator is firstly an entrant and then becomes an incumbent whose patent is infringed upon. Therefore, setting aside the issues of profit growth and discounting, every innovator receives the same amount of profit over the lifetime of an invention. Given that the real interest rate is higher than the growth rate in their model, delaying the receipt of profits reduces the present value of the

[5] See for example Jaffe and Lerner (2004) and Bessen and Meurer (2008). Boldrin and Levine (2008) refer to patents as ''intellectual monopoly'' and argue for the elimination of all patents.

[6] See for example Green and Scotchmer (1995) and Gallini and Scotchmer (2002) for a discussion on the importance of this policy lever.

income stream. As a result, the complete frontloading profit-division rule (i.e., allowing the entrant to keep all the profit) tends to maximize the market value of an invention and the incentives for R&D.[7] However, in the present study with both vertical and horizontal innovation, this result no longer holds. In this case, the inventor of a new variety is the first innovator on a quality ladder; therefore, assigning a larger share of profit to the incumbent increases horizontal innovation. Given that quality improvement and variety expansion are both important channels for economic growth, the growth-maximizing profit-division rule should balance between the asymmetric effects of profit division on vertical and horizontal innovation. Furthermore, given that growth maximization does not necessarily give rise to welfare maximization, we characterize both the growth-maximizing and welfare-maximizing profit-division rules.

This study also relates to other growth-theoretic studies on patent policy. Judd (1985) provides the seminal dynamic general-equilibrium analysis on patent length, and he finds that an infinite patent length maximizes innovation and welfare. Subsequent studies find that strengthening patent protection in various forms does not necessarily increase innovation and may even stifle it. Examples include Horowitz and Lai (1996) on patent length, O'Donoghue and Zweimuller (2004) on leading breadth and patentability requirement, Koleda (2004) on patentability requirement, and Furukawa (2007) and Horii and Iwaisako (2007) on patent protection against imitation. The present study differs from these studies by (a) analyzing a different patent-policy lever (i.e., the profit-division rule between sequential innovators) and (b) emphasizing the asymmetric effects of patent protection on vertical and horizontal innovation.[8] In other words, rather than analyzing the effects of patent policy on the *level* of innovation as is common in the literature, we consider a much less explored question that is the effects of patent policy on the *composition* or *direction* of innovation. A recent study by Iwaisako and Futagami (in press) examines the contrasting effects of patent breadth on innovation and physical capital accumulation, and they also show that the relationship between patent protection and economic growth may follow an inverted-U shape.

Cozzi (2001) analyzes patent protection in the form of intellectual appropriability (i.e., the ability of an innovator to patent her invention in the presence of spying activities) in a quality-ladder model. Cozzi and Spinesi (2006) extend this analysis into a model with both vertical and horizontal innovation. In their model, spying activities are targeted only at quality improvement. Therefore, strengthening intellectual appropriability stimulates vertical innovation (at the expense of horizontal innovation) and increases long-run growth because horizontal innovation only has a level effect in their model for removing scale effects. Eicher and Garcia-Peñalosa (2008) consider endogenous mis-appropriation by endogenizing firm level costly institution building activities to counter-piracy, in an economy where horizontal innovation is the engine of growth.[9] In contrast, in the present study, long-run growth depends on both vertical and horizontal innovation,[10] and hence, the asymmetric effects of profit division on vertical and horizontal innovation give rise to a growth-maximizing profit-division rule.

Acs and Sanders (in press) and Cozzi and Galli (2011) also analyze the division of profit between innovators. Acs and Sanders (in press) analyze the separation between invention and commercialization in a variety-expanding model whereas Cozzi and Galli (2011) consider basic research and applied research in a quality-ladder model. In these studies, each invention (i.e., a new variety or a quality improvement) is created in a two-step innovation process; therefore, there exists a growth-maximizing division of profit that balances between the incentives of the first and second innovators of each invention. The present study differs from these studies by analyzing the division of profit between sequential innovators within the same industry (in which every innovator is firstly an entrant and then becomes an incumbent). Also, we consider a model that features both vertical and horizontal innovation. We find that frontloading (backloading) the income stream along the quality ladder stimulates vertical (horizontal) innovation, and it is the interaction between these two types of innovation that gives rise to a growth-maximizing profit-division rule in this study.

This study also relates to Acemoglu (2009), who shows that under the current patent system, the equilibrium diversity of innovation is insufficient. In other words, innovators have too much incentive to invest in R&D on improving existing products but too little incentive to invest in R&D on developing new products that may become useful in the future. Acemoglu suggests that increasing the diversity of researchers could be a partial remedy against this problem of insufficient diversity. The present study suggests another possible solution that is to increase the share of profit assigned to the pioneering inventor of a product. In this case, there will be a reallocation of research inputs from vertical innovation (i.e., R&D on existing products) to horizontal innovation (i.e., R&D on new products).

The rest of this study is organized as follows. Section 2 describes the model. Section 3 defines the equilibrium and characterizes the equilibrium allocation. Section 4 considers the growth and welfare effects of patents. Section 5 calibrates the model and simulates transition dynamics. Section 6 considers two extensions of the model. The final section concludes.

[7] See also Chu (2009) for a quantitative analysis of the profit-division rule in the O'Donoghue–Zweimuller model.

[8] O'Donoghue and Zweimuller (2004) also consider a model with both vertical and horizontal innovation in their appendix. However, their focus is on the effects of patentability requirement and leading breadth, and they did not analyze the effects of alternative profit-division rules in the presence of vertical and horizontal innovation.

[9] Interestingly, they also detect multiple institutional equilibria, provided that the scale of the economy is large enough.

[10] See footnotes 11 and 29 for a discussion on the issue of scale effects in R&D-based growth models.

2. The model

To consider both vertical and horizontal innovation in an R&D-based growth model,[11] we modify the Grossman and Helpman (1991) quality-ladder model[12] by endogenizing the number of varieties in the economy. Furthermore, to consider the division of profit between sequential innovators along the quality ladder, we assume that each entrant (i.e., the most recent innovator) infringes the patent of the incumbent (i.e., the previous innovator). As a result of this patent infringement, the entrant has to transfer a share $s \in [0,1]$ of her profit to the incumbent. However, with vertical innovation, every innovator's patent would eventually be infringed by the next innovation, and she can then extract a share s of profit from the next entrant. This formulation of profit division between sequential innovators originates from O'Donoghue and Zweimuller (2004). As for horizontal innovation, the invention of a new variety does not infringe any patent,[13] so that a variety inventor does not have to share her profit but maintains the rights to extract profit from the next entrant. Given that the Grossman–Helpman model is well-studied, we will describe the familiar features briefly to conserve space and discuss new features (i.e., variety expansion and profit division) in details.

2.1. Households

There is a unit continuum of identical households. Their lifetime utility is given by

$$U = \int_0^\infty e^{-\rho t} \ln c_t \, dt, \tag{1}$$

where $\rho > 0$ is discount rate, and c_t is the consumption index at time t. The consumption index is defined as[14]

$$c_t \equiv \exp\left(\int_0^{n_t^*} \ln y_t(i) \, di \right). \tag{2}$$

(2) shows that the households derive utility by consuming a continuum of products $y_t(i)$. In Grossman and Helpman (1991), there is a unit continuum of these products. In the present study, we endogenize the number of varieties by allowing for horizontal innovation. n_t^* is the number of active varieties that are consumed by households at time t, and its law of motion is given by

$$\dot{n}_t^* = \dot{n}_t - \delta n_t^*. \tag{3}$$

n_t is the total number of varieties that have been invented in the past, and $\dot{n}_t$ is the number of newly invented varieties at time t. We follow Grossman and Lai (2004) to allow for the possibility that an invented variety becomes obsolete at some point. For tractability, we assume that each active variety $i \in [0, n_t^*]$ at time t faces the same probability $\delta > 0$ to become permanently obsolete.[15]

Households maximize (1) subject to

$$\dot{a}_t = r_t a_t + w_{h,t} + w_{l,t} L - \int_0^{n_t^*} p_t(i) y_t(i) \, di. \tag{4}$$

a_t is the value of assets owned by households, and r_t is the rate of return. To simplify the analysis, we assume that households supply one unit of high-skill labor for R&D and $L > 1$ units of low-skill labor for production.[16] The wage rates for high-skill and low-skill labors are $w_{h,t}$ and $w_{l,t}$ respectively. $p_t(i)$ is the price of product i at time t. If we denote ζ_t as the Hamiltonian co-state variable, then households' intratemporal optimality condition is

$$p_t(i) y_t(i) = 1/\zeta_t, \tag{5}$$

for $i \in [0, n_t^*]$, and the intertemporal optimality condition is

$$r_t = \rho - \dot{\zeta}_t / \zeta_t. \tag{6}$$

[11] See also Dinopoulos and Thompson (1999a,b), Howitt (1999), Jones (1999), Li (2000), Peretto (1998, 1999), Peretto and Smulders (2002), Segerstrom (2000) and Young (1998). The focus of these studies is on the removal of scale effects in R&D-based growth models. Given that scale effect is not the focus of this study, we normalize the supply of skilled labor to unity to set aside this issue.

[12] See also Aghion and Howitt (1992) and Segerstrom et al. (1990) for other pioneering studies on the quality-ladder growth model.

[13] In the main text, we also discuss the alternative case in which a newly invented variety infringes the patents of other existing varieties.

[14] In their appendix, O'Donoghue and Zweimuller (2004) also consider this Cobb–Douglas specification, which is similar to the CES form in Howitt (1999) and Segerstrom (2000) except for the different elasticity of substitution across varieties. In this study, we focus on the Cobb–Douglas aggregator which enables us to compute the consumption index's transition path along which the arrival rate of innovation varies.

[15] Due to the quality distribution across varieties, the model would become considerably more complicated if we allow the obsolescence rate to depend on the age of a variety.

[16] In Grossman and Helpman (1991), a homogeneous type of labor is allocated between R&D and production. In reality, R&D engineers and scientists often have a high level of education. Given that this model features two R&D sectors involving the allocation of high-skill labor, we naturally distinguish between high-skill labor for R&D and low-skill labor for production. However, it is useful to note that our main results carry over to a setting with homogeneous labor that is allocated across production, vertical R&D and horizontal R&D; see Section 6.2 for this extension.

2.2. Production

There is a continuum of active varieties $i \in [0, n_t^*]$ that are consumed by households at time t. The production function for the most recent innovator in industry i is

$$y_t(i) = z^{q_t(i)} l_t(i). \tag{7}$$

The parameter $z > 1$ is the exogenous step size of each productivity improvement. $q_t(i)$ is the number of productivity improvements that have occurred in industry i as of time t. $l_t(i)$ is the number of low-skill production workers employed in industry i. Given $z^{q_t(i)}$, the marginal cost of production for the most recent innovator in industry i is

$$mc_t(i) = w_{l,t}/z^{q_t(i)}. \tag{8}$$

Notice that we here adopt a "cost reducing" view of vertical innovation following Peretto (1998, 1999) and Peretto and Smulders (2002).[17] In each industry that has at least two generations of innovation, the most recent innovator infringes the previous innovator's patent. As a result of this patent infringement, the most recent innovator pays a licensing fee by transferring a share s of her profit to the previous innovator. We follow O'Donoghue and Zweimuller (2004) to consider an exogenous profit-division rule.[18,19] This profit-division rule can be interpreted as the outcome of a bargaining game, in which the bargaining power of each side can be influenced by patent policy.[20] Therefore, it is not an unrealistic assumption to treat s as a policy parameter.

O'Donoghue and Zweimuller (2004) are interested in the effects of leading breadth on R&D and economic growth through the consolidation of market power that enables the most recent innovator and the previous innovator to consolidate their market power and charge a higher markup. We do not adopt this formulation here for three reasons. First, the collusion between innovators may be prohibited by antitrust laws. Second, the licensing agreement only allows the most recent innovator to produce, but it may not prevent the previous innovator from selling her products at a lower price. As a result, the previous innovator may have the incentives to continue selling her products and undercut the markup. Third, we want to focus on the profit-division effect (instead of the markup effect) of patent protection in this study. Given these considerations, we assume that the most recent innovator and the previous innovator engage in the usual Bertrand competition as in Grossman and Helpman (1991). The profit-maximizing price for the most recent innovator is a constant markup (given by the step size z) over her own marginal cost in (8).[21]

$$p_t(i) = z(w_{l,t}/z^{q_t(i)}). \tag{9}$$

Given (7)–(9), the monopolistic profit generated by the most recent innovation is

$$\pi_t(i) = (z-1)w_{l,t} l_t(i) = \left(\frac{z-1}{z}\right)\frac{1}{\zeta_t}, \tag{10}$$

where the second equality is obtained by using (5), (7) and (9). Due to profit division, the most recent innovator obtains $(1-s)\pi_t$ while the previous innovator obtains $s\pi_t$. The above discussion implicitly assumes that the most recent innovation and the second-most recent innovation are owned by different firms (i.e., the Arrow replacement effect). In Lemma 1, we show that the Arrow replacement effect is indeed present in this quality-ladder model with profit division.[22]

Lemma 1. *The Arrow replacement effect is present.*

Proof. See Appendix A. □

Finally, for a newly invented variety, we make the usual simplifying assumption that the productivity of labor in each new variety[23] is randomly drawn from the existing distribution of active products $i \in [0, n_t^*]$. We also assume that a variety

[17] It is useful to note that cost reduction is isomorphic to quality improvement in these studies as well as in the current framework. To see this, the reader could easily reinterpret $y_t(i)$ as the consumption of the latest version, $q_t(i)$, of product i, along the lines of Grossman and Helpman (1991), that is by assuming $\ln c_t \equiv (\int_0^{n_t^*} \ln \sum_{j=0}^{q_t(i)} z^j y_t(i)\, di)$, with consumption good i's production function given by $y_t(i) = l_t(i)$. Clearly, the profit function (10) would follow directly from Bertrand competition, instead of the no longer valid (8) and (9).

[18] O'Donoghue and Zweimuller (2004) consider the more general case in which the current innovator may infringe the patents of multiple previous innovators. For the purpose of the present study, it is sufficient to demonstrate the asymmetric effects of the profit-division rule on vertical and horizontal innovation by considering the simple case of profit division between the entrant and the incumbent.

[19] Chu and Pan (in press) analyze the effects of blocking patents under the case of an *endogenous* profit-division rule and an *endogenous* step size of innovation in a quality-ladder model with only vertical innovation. As in the present study, they also find that blocking patents have a non-monotonic effect on economic growth.

[20] In reality, a patentholder enforces her patent rights through the Court, which decides her case of patent infringement against a potential infringer. Therefore, when it becomes more likely for the Court to favor patentees, the bargaining power of patentholders strengthens relative to potential infringers. Of course, this will indirectly affect also the outcomes of potential pre-trial settlements.

[21] Li (2001) considers a CES version of (2) without horizontal innovation. In this case, the monopolistic markup is determined by either the quality step size or the elasticity of substitution depending on whether innovation is drastic or non-drastic. Without loss of generality, we focus on non-drastic innovation as in the original Grossman–Helpman model.

[22] Cozzi (2007) shows that the Arrow effect is not necessarily inconsistent with the empirical observation that incumbents often target innovation at their own industries. Under this interpretation, the incumbents' choice of R&D is simply indeterminate, so that the aggregate economy behaves *as if* innovation is targeted only by entrants. See also Etro (2004, 2008) for an interesting analysis on innovation by incumbents with a first-mover advantage.

[23] Or the quality of each new variety, in the equivalent quality ladder interpretation explained above.

inventor can only patent the most advanced technology. Given that the lower-productivity production methods are unpatented, Bertrand competition drives the markup down to z as well.[24] However, because there is no previous patentholder in the newly created industry, the variety inventor obtains the entire π until the next productivity improvement occurs, and then she can extract $s\pi$ from the entrant.

What happens when a variety invention infringes the patents of existing varieties? For example, Hall et al. (2001) define an original innovation as "a patent that cites a broad set of technologies or which has a certain percentage of citations given to different patent classes". If we view an original innovation as a horizontal innovation and assume that the probability of patent infringement is increasing in the number of patent citations, then horizontal innovation may in fact be more at risk of patent infringements. Here we discuss the implication of an alternative assumption that a newly invented variety infringes all previous horizontal patents. In this case, the infringed patentholders should all claim a right to share among themselves a fraction of the profits. But this means that the share that will go to each infringed patentholder is zero as a result of the continuum of products (or tending to zero with countable products). Let us assume that each infringed party has to pay a however small, but discrete, legal fee ε in order to sue the infringer. Then, in equilibrium no previous horizontal innovator will ever sue the current horizontal innovator.

2.3. *Vertical innovation*

Denote $v_{2,t}(i)$ as the value of the patent held by the second-most recent innovator in industry i. Because $\pi_t(i)=\pi_t$ for $i \in [0,n_t^*]$ from (10), $v_{2,t}(i)=v_{2,t}$ in a symmetric equilibrium (i.e., an equal arrival rate of innovation across industries).[25] In this case, the familiar no-arbitrage condition for $v_{2,t}$ is

$$r_t v_{2,t} = s\pi_t + \dot{v}_{2,t} - (\delta+\lambda_t)v_{2,t}. \tag{11}$$

The left-hand side of (11) is the return on this asset. The right-hand side of (11) is the sum of (a) the profit $s\pi_t$ received by the patentholder, (b) the potential capital gain $\dot{v}_{2,t}$, and (c) the expected capital loss due to obsolescence $\delta v_{2,t}$ and creative destruction $\lambda_t v_{2,t}$, where λ_t is the Poisson arrival rate of innovation in the industry. As for the value of the patent held by the most recent innovator, the no-arbitrage condition for $v_{1,t}$ is

$$r_t v_{1,t} = (1-s)\pi_t + \dot{v}_{1,t} - (\delta+\lambda_t)v_{1,t} + \lambda_t v_{2,t}. \tag{12}$$

The intuition behind (12) is the same as (11) except for the addition of the last term. When the next quality improvement occurs, the most recent innovator becomes the second-most recent innovator, and hence, her net expected capital loss is $\lambda_t(v_{1,t}-v_{2,t})$.

There is a unit continuum of vertical-R&D firms indexed by $j \in [0,1]$ doing research on vertical innovation in each industry i. They hire high-skill labor $h_{q,t}(j)$ to create productivity improvements, and the expected profit of firm j is

$$\pi_{q,t}(j) = v_{1,t}\lambda_t(j) - w_{h,t}h_{q,t}(j). \tag{13}$$

The *firm-level* arrival rate of innovation is

$$\lambda_t(j) = \bar{\varphi}_{q,t}h_{q,t}(j), \tag{14}$$

where $\bar{\varphi}_{q,t}$ is the productivity of vertical R&D at time t. The zero-expected-profit condition for vertical R&D is

$$v_{1,t}\bar{\varphi}_{q,t} = w_{h,t}. \tag{15}$$

We follow Jones and Williams (2000) to assume that $\bar{\varphi}_{q,t} = \varphi_q(h_{q,t})^{\phi_q-1}$, where $\varphi_q > 0$ is a productivity parameter for vertical R&D and $\phi_q \in (0,1)$ captures the usual negative externality in intratemporal duplication within each industry. In equilibrium, the industry-level arrival rate of innovation equals the aggregate of firm-level arrival rates. Therefore, at the *aggregate* level, the arrival rate of vertical innovation for each variety is $\lambda_t = \varphi_q(h_{q,t})^{\phi_q}$, which is subject to decreasing returns to scale[26]; see for example Kortum (1993) and Thompson (1996) for empirical evidence.

2.4. *Horizontal innovation*

Denote $v_{n,t}$ as the value of inventing a new variety. The no-arbitrage condition for $v_{n,t}$ is

$$r_t v_{n,t} = \pi_t + \dot{v}_{n,t} - (\delta+\lambda_t)v_{n,t} + \lambda_t v_{2,t}. \tag{16}$$

The only difference between (12) and (16) is that a variety inventor captures π_t while a quality innovator captures $(1-s)\pi_t$. There is also a unit continuum of horizontal-R&D firms indexed by $k \in [0,1]$ doing research on creating new varieties. They

[24] In the alternative case of drastic innovation, a new variety inventor and the most recent innovator for an existing variety would also choose the same equilibrium markup that is determined by the elasticity of substitution.

[25] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi (2005) and Cozzi et al. (2007) for a discussion on the symmetric equilibrium in the quality-ladder model.

[26] Despite decreasing returns to scale at the aggregate level, we assume constant returns to scale at the firm level in order to be consistent with free entry and zero expected profit in the R&D sector.

hire high-skill labor $h_{n,t}(k)$ to create inventions, and the profit of firm k is

$$\pi_{n,t}(k) = v_{n,t}\dot{n}_t(k) - w_{h,t}h_{n,t}(k). \tag{17}$$

The number of inventions created by firm k is[27]

$$\dot{n}_t(k) = \bar{\varphi}_{n,t}h_{n,t}(k), \tag{18}$$

where $\bar{\varphi}_{n,t}$ is the productivity of horizontal R&D at time t. The zero-profit condition for horizontal R&D is

$$v_{n,t}\bar{\varphi}_{n,t} = w_{h,t}. \tag{19}$$

Again, $\bar{\varphi}_{n,t} = \varphi_n(h_{n,t})^{\phi_n - 1}$, where $\varphi_n > 0$ is a productivity parameter for variety-expanding R&D and $\phi_n \in (0,1)$ captures the duplication externality in horizontal innovation. At the *aggregate* level, the total number of inventions created at time t is

$$\dot{n}_t = \varphi_n(h_{n,t})^{\phi_n}. \tag{20}$$

3. Decentralized equilibrium

The equilibrium is a time path $\{y_t(i), l_t, h_{q,t}, h_{n,t}, r_t, p_t(i), w_{l,t}, w_{h,t}, v_{n,t}, v_{1,t}, v_{2,t}\}$, $t \geq 0$. Also, at each instant of time,

- households maximize utility taking $\{r_t, p_t(i), w_{l,t}, w_{h,t}\}$ as given;
- production firms produce $\{y_t(i)\}$ and choose $\{p_t(i)\}$ to maximize profit taking $\{w_{l,t}\}$ as given;
- vertical-innovation firms choose $\{h_{q,t}\}$ to maximize expected profit taking $\{w_{h,t}, v_{1,t}\}$ as given;
- horizontal-innovation firms choose $\{h_{n,t}\}$ to maximize profit taking $\{w_{h,t}, v_{n,t}\}$ as given;
- the low-skill labor market clears such that $n_t^* l_t = L$; and
- the high-skill labor market clears such that $h_{n,t} + n_t^* h_{q,t} = 1$.

3.1. Stationary equilibrium

We focus on a stationary equilibrium, in which the number of active varieties is constant. Substituting (20) into (3) yields $\dot{n}_t^* = \varphi_n(h_{n,t})^{\phi_n} - \delta n_t^*$. Therefore, $\dot{n}_t^* = 0$ implies that

$$n^* = \dot{n}/\delta = \varphi_n(h_n)^{\phi_n}/\delta. \tag{21}$$

The number of production workers per variety is

$$l = \frac{L}{n^*} = \frac{\delta L}{\varphi_n(h_n)^{\phi_n}}. \tag{22}$$

Let us choose low-skill labor as the numeraire (i.e., $w_{l,t} = 1$ for all t). Then, combining (5), (7) and (9) shows that ζ is constant in the stationary equilibrium implying that $r = \rho$ from (6) and $\dot{\pi}_t/\pi_t = 0$ from (10). Applying the stationary equilibrium conditions on (11), (12) and (16) yields

$$v_1 = \frac{(1-s)\pi + \lambda v_2}{\rho + \delta + \lambda} = \frac{\pi}{\rho + \delta + \lambda}\left(1 - s + s\frac{\lambda}{\rho + \delta + \lambda}\right), \tag{23}$$

$$v_n = \frac{\pi + \lambda v_2}{\rho + \delta + \lambda} = \frac{\pi}{\rho + \delta + \lambda}\left(1 + s\frac{\lambda}{\rho + \delta + \lambda}\right). \tag{24}$$

(24) shows that the value of a new variety v_n is increasing in s for a given innovation rate λ because a larger s allows the variety inventor to extract more profit from the next innovator. In contrast, (23) shows that the value of a productivity improvement v_1 is decreasing in s for a given λ because of the backloading effect $\lambda/(\rho + \delta + \lambda) < 1$. In other words, delaying the income stream reduces its expected present value due to discounting ρ and the possibility of obsolescence δ.

Substituting (23) and (24) into $v_1\bar{\varphi}_q = v_n\bar{\varphi}_n$ from (15) and (19) yields

$$(h_n)^{1-\phi_n} = \left(\frac{\varphi_n}{\varphi_q}\frac{\rho + \delta + (1+s)\varphi_q(h_q)^{\phi_q}}{(1-s)(\rho + \delta) + \varphi_q(h_q)^{\phi_q}}\right)(h_q)^{1-\phi_q}. \tag{25}$$

We will refer to (25) as the *arbitrage condition*. To close the model, we manipulate $h_{n,t} + n_t^* h_{q,t} = 1$ to derive

$$\frac{\delta(1-h_n)}{\varphi_n(h_n)^{\phi_n}} = h_q. \tag{26}$$

[27] Due to the assumption of a continuum of varieties, there is no strategic interaction across varieties. Therefore, we do not need to distinguish between single-product and multi-product firms.

We will refer to (26) as the *resource constraint*. The equilibrium allocation of high-skill labor is implicitly determined by solving (25) and (26). Taking the total differentials of (26) yields

$$\frac{dh_n}{dh_q} = -\left(\frac{1-h_n}{h_n+\phi_n(1-h_n)}\right)\frac{h_n}{h_q} < 0. \tag{27}$$

In other words, the resource constraint describes a negative relationship between h_n and h_q. As for the arbitrage condition in (25), h_q has opposing effects on the arbitrage condition. On the one hand, an increase in h_q decreases $\bar{\varphi}_q$. For a given value of v_n/v_1, h_n must rise and $\bar{\varphi}_n$ must fall to balance $v_1\bar{\varphi}_q = v_n\bar{\varphi}_n$. On the other hand, a larger h_q increases λ and decreases v_n/v_1 when $s>0$. If this latter effect is strong enough, it may lead to a decrease in h_n. Taking the total differentials of (25) yields

$$\frac{dh_n}{dh_q} = \frac{1}{1-\phi_n}\left(1-\phi_q-\phi_q\frac{s^2(\rho+\delta)}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}}\frac{\varphi_q(h_q)^{\phi_q}}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}\right)\frac{h_n}{h_q}. \tag{28}$$

(28) shows that dh_n/dh_q must be positive when h_q equals zero or becomes sufficiently large. However, at intermediate values of h_q, it is possible for dh_n/dh_q to be negative. In this case, there may be multiple equilibria. To rule out multiple equilibrium, which is not the focus of this study, Lemma 2 derives the parameter condition under which (28) is always positive, which is sufficient to ensure that the stationary equilibrium is unique. Lets define a parameter threshold $\overline{\phi}_q \equiv [1-0.5s^2/(1+\sqrt{1-s^2})] \in [0.5,1]$.

Lemma 2. *If $\phi_q < \overline{\phi}_q$, then $dh_n/dh_q > 0$ in (28) $\forall h_q > 0$.*

Proof. See Appendix A □.

Fig. 1 plots (25) and (26) in the (h_q,h_n) space. The resource constraint (RC) is negatively sloped while the arbitrage condition (AC) is positively sloped given the parameter condition in Lemma 2.

Therefore, if an equilibrium exists, it must be unique. Also, a larger s increases the market value of a new variety and decreases that of a quality improvement; consequently, horizontal R&D h_n rises and vertical R&D h_q falls. Given this intuitive result (summarized in Proposition 1), the next section uses the growth-theoretic framework to analyze the effects of the profit-division rule on economic growth and social welfare.

Proposition 1. *Given $\phi_q < \overline{\phi}_q$, there exists a unique equilibrium (h_q,h_n). The equilibrium $h_n(s)$ is increasing in s whereas $h_q(s)$ is decreasing in s.*

Proof. At $h_q=0$, $h_n=0$ in (25) and $h_n=1$ in (26). As h_q approaches infinity, h_n in (26) approaches zero. Therefore, (25) and (26) must cross exactly once given Lemma 2. An increase in s shifts up (25) in the (h_q,h_n) space leading to an increase in h_n and a decrease in h_q. See Fig. 1. □

4. Growth and welfare effects of blocking patents

In this section, we analyze the effects of blocking patents on economic growth and social welfare. We first derive the growth-maximizing profit-division rule and then the welfare-maximizing rule. Finally, we compare them and characterize the condition under which one is above the other.

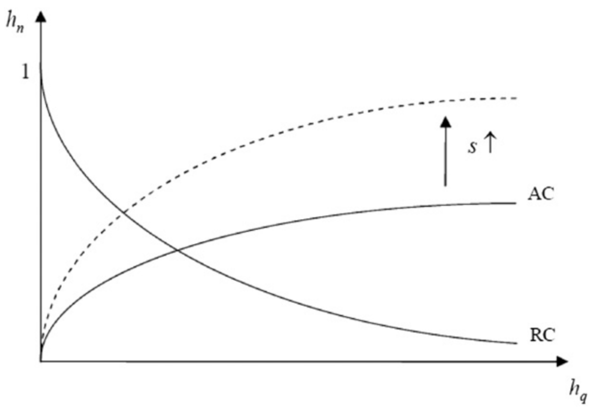

Fig. 1. Stationary equilibrium.

4.1. The growth-maximizing profit-division rule

To derive the balanced growth rate of the consumption index, we substitute (7) into (2) to obtain

$$\ln c_t = \int_0^{n^*} [q_t(i)\ln z + \ln l(i)]di = \left(n^* \int_0^t \lambda_\tau \, d\tau\right)\ln z + n^* \ln l. \tag{29}$$

The second equality of (29) is obtained by (a) applying symmetry $l(i) = l$ from (10), (b) normalizing $q_0(i) = 0$ for all i, and (c) using the law of large numbers that implies $\int_0^{n^*} q_t(i)\,di = n^* \int_0^t \lambda_\tau \, d\tau$.[28] Differentiating (29) with respect to time yields the balanced growth rate of the consumption index given by

$$g \equiv \frac{\dot{c}_t}{c_t} = n^* \lambda \ln z, \tag{30}$$

where the steady-state number of varieties is $n^* = \varphi_n (h_n)^{\phi_n}/\delta$, and the arrival rate of productivity improvement in each industry is $\lambda = \varphi_q (h_q)^{\phi_q}$.

Corollary 1. *n^* is increasing in s whereas λ is decreasing in s.*

Proof. Recall that $n^* = \varphi_n (h_n)^{\phi_n}/\delta$ and $\lambda = \varphi_q (h_q)^{\phi_q}$. Then, from Proposition 1, h_n is increasing in s whereas h_q is decreasing in s. □

To see why the equilibrium growth rate depends on the number of varieties, lets consider the symmetric case of (2) given by $\ln c_t = n^* \ln y_t(i)$. Differentiating $\ln c_t$ with respect to time yields $g = n^* \dot{y}_t(i)/y_t(i)$. In other words, for a given quality growth rate of each variety, increasing the number of varieties causes the aggregate consumption index to grow at a higher rate.[29] Given that increasing s has a positive effect on n^* and a negative effect on λ, there is generally a growth-maximizing profit-division rule. Differentiating the log of (30) with respect to s yields

$$\frac{1}{g}\frac{\partial g}{\partial s} = \frac{\phi_n}{h_n}\frac{\partial h_n}{\partial s} + \frac{\phi_q}{h_q}\frac{\partial h_q}{\partial s}, \tag{31}$$

where $\partial h_n/\partial s > 0$ and $\partial h_q/\partial s < 0$ from Proposition 1. From (27), we can derive

$$\frac{1}{h_n}\frac{dh_n}{ds} = -\frac{1}{h_q}\left(\frac{1-h_n}{h_n + \phi_n(1-h_n)}\right)\frac{dh_q}{ds}. \tag{32}$$

Substituting (32) into (31) yields

$$\frac{1}{g}\frac{\partial g}{\partial s} = -\frac{1}{h_q}\left(\frac{\phi_n(1-h_n)}{h_n + \phi_n(1-h_n)} - \phi_q\right)\frac{dh_q}{ds}. \tag{33}$$

Therefore,

$$\frac{\partial g}{\partial s} > 0 \Leftrightarrow h_n(s) < \Phi \equiv \frac{\phi_n(1-\phi_q)}{\phi_q + \phi_n(1-\phi_q)}. \tag{34}$$

To gain a better understanding of (34), we maximize (30) by directly choosing h_n and h_q subject to (26). Substituting $\lambda = \varphi_q (h_q)^{\phi_q}$ and $h_q = (1-h_n)/n^*$ into (30) yields $g = (n^*)^{1-\phi_q}(1-h_n)^{\phi_q}\varphi_q \ln z$, where $n^* = \varphi_n (h_n)^{\phi_n}/\delta$ from (21). It is easy to show that the growth-maximizing h_n is given by Φ, which is increasing in ϕ_n and decreasing in ϕ_q. In other words, as horizontal R&D exhibits a smaller degree of negative duplication externality (i.e., a larger ϕ_n) or as vertical R&D exhibits a larger degree of duplication externality (i.e., a smaller ϕ_q), the economy should allocate more research labor to horizontal R&D to maximize economic growth. Therefore, the growth-maximizing profit-division rule $s_g \equiv \arg\max g(s)$ is characterized by moving the equilibrium $h_n(s_g)$ to as close to Φ as possible.

Proposition 2. *If an interior growth-maximizing profit-division rule s_g exists, it is implicitly defined by $h_n(s_g) = \Phi$. If $h_n(0) > \Phi$, then $s_g = 0$. If $h_n(1) < \Phi$, then $s_g = 1$.*

[28] Note that at each instant of time, the average quality of new varieties is the same as the average quality of obsolete varieties because they are drawn from the same quality distribution. In Appendix B, we derive an expression for $\ln c_t$ when n_t^* varies over time.

[29] It is useful to note that this result of horizontal innovation affecting long-run growth does not rely on a stationary number of varieties. In the case of a growing number of varieties, horizontal innovation would still have an effect on long-run growth if the long-run variety growth rate is endogenous. However, it is common for studies on R&D-based growth models with vertical and horizontal innovation to assume a setup in which the long-run variety growth rate is equal to the exogenous population growth rate for the purpose of eliminating scale effects.

Proof. Note (33) and (34). Also, recall that $h_n(s)$ is increasing in s. □

4.2. The welfare-maximizing profit-division rule

To derive the steady-state welfare,[30] we normalize the time index such that time 0 is the instant when the economy reaches the stationary equilibrium. In this case, (1) becomes[31]

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho}\right) = \frac{1}{\rho}\left(n^* \ln l + \frac{n^* \lambda \ln z}{\rho}\right), \tag{35}$$

where $l = L/n^*$ is decreasing in s. In other words, social welfare is determined by the growth rate g as well as the initial level of consumption $\ln c_0$. Because of this additional level effect, the welfare-maximizing profit-division rule is generally different from the growth-maximizing rule. When s increases, it creates a positive effect as well as a negative effect on $\ln c_0 = n^* \ln l$. By increasing h_n and n^*, a larger s increases the number of varieties available for consumption on the one hand and decreases output per variety on the other. Differentiating $\ln c_0$ with respect to s yields

$$\frac{\partial \ln c_0}{\partial s} = (\ln l - 1)\frac{\partial n^*}{\partial s}, \tag{36}$$

where $n^* = \varphi_n (h_n)^{\phi_n}/\delta$ so that $\partial n^*/\partial s > 0$. Therefore,

$$\frac{\partial \ln c_0}{\partial s} > 0 \Leftrightarrow h_n(s) < \Delta \equiv \left(\frac{\delta L}{\varphi_n e}\right)^{1/\phi_n}, \tag{37}$$

where $e = \exp(1)$. In other words, the level of h_n that maximizes initial consumption is given by Δ. Eq. (22) shows that for a given $(h_n)^{\phi_n}$, a larger $\delta L/\varphi_n$ increases l, so that h_n can be larger while initial consumption still rises.

Differentiating (35) with respect to s yields

$$\frac{\partial U}{\partial s} = \frac{1}{\rho}\left(\frac{\partial \ln c_0}{\partial s} + \frac{1}{\rho}\frac{\partial g}{\partial s}\right). \tag{38}$$

Denote the welfare-maximizing profit-division rule by $s_u \equiv \arg\max U(s)$.[32] In Proposition 3, we show that

$$s_u \geq s_g \Leftrightarrow \Delta \geq \Phi. \tag{39}$$

Intuitively, the welfare-maximizing h_n balances between the growth effect and the initial-level effect on welfare. Therefore, it is a weighted average of Δ and Φ. If $\Delta \geq \Phi$, then the welfare-maximizing h_n is above the growth-maximizing h_n, and vice versa. Given that $h_n(s)$ is increasing in s, $\Delta \geq \Phi$ would also imply $s_u \geq s_g$.

Proposition 3. *The welfare-maximizing profit-division rule s_u is below (above) the growth-maximizing profit-division rule s_g if Δ is smaller (larger) than Φ.*

Proof. From (34), we know that $\partial g/\partial s = 0$ at $h_n(s) = \Phi$. From (37), we know that $\partial \ln c_0/\partial s = 0$ at $h_n(s) = \Delta$. Suppose $\Delta = \Phi$. Then, (38) shows that $s_u = s_g$. If $\Delta \geq (\leq)\Phi$, then $s_u \geq (\leq) s_g$ because $h_n(s)$ is increasing in s. □

Finally, we discuss how the supply of unskilled labor L affects the welfare-maximizing profit-division rule. From (25) and (26), we see that neither the arbitrage condition nor the resource constraint depend on L. Therefore, the supply of unskilled labor has no effect on the growth-maximizing profit-division rule. Furthermore, given that Δ is increasing in L, it must be the case that s_u is increasing in L. Intuitively, a larger supply of unskilled labor increases output per variety and magnifies the positive effect of n^* on the initial level of consumption $\ln c_0 = n^* \ln L - n^* \ln n^*$ through the term $n^* \ln L$. Given that the welfare-maximizing s_u is increasing in L while the growth-maximizing s_g is independent of L, we have the following result illustrated in Fig. 2, in which we define a threshold value of L given by $\overline{L} \equiv \varphi_n \Phi^{\phi_n} e/\delta$.

Corollary 2. *If L is smaller (larger) than $\overline{L}$, then s_u is below (above) s_g.*

Proof. This result follows from Proposition 3 because $L \leq \overline{L} \equiv \varphi_n \Phi^{\phi_n} e/\delta$ is equivalent to $\Delta \leq \Phi$. □

[30] In this section, we restrict our attention to steady-state welfare. A more complete welfare analysis would take into account the evolution of households' utility during the transitional path from the initial state to the steady state, and we will perform this analysis numerically in the next section. However, such an analysis is analytically much more complicated. Therefore, we first follow the usual treatment in the literature to derive the optimal patent policy that maximizes steady-state welfare. See for example Iwaisako and Futagami (2003), Grossman and Lai (2004), Futagami and Iwaisako (2007) and Acemoglu and Akcigit (2012).

[31] Eq. (35) is based on the normalization that $q_0(i) = 0$ for all i. If we modify this normalization to $q_0(i) = q > 0$ for all i, then there will be an extra term $n^* q \ln z$ inside the bracket in (35). It can be shown that $q > 0$ has the same effect as a larger L on steady-state welfare.

[32] It is useful to note that as in the case of the growth-maximizing profit-division rule, the welfare-maximizing profit-division rule can be a corner solution (i.e., $s_u = 0$ or $s_u = 1$).

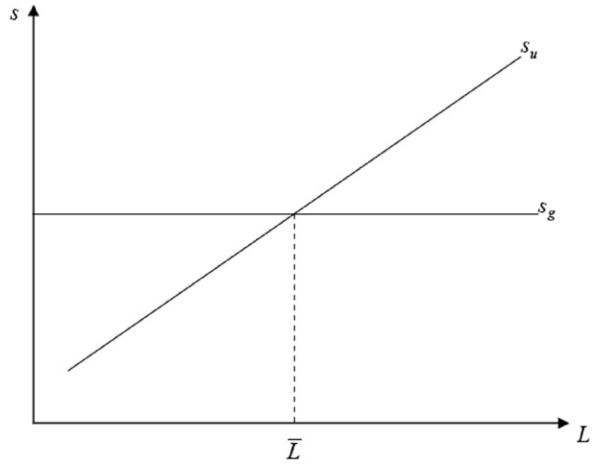

Fig. 2. Growth-maximizing and welfare-maximizing profit-division rules.

5. Quantitative analysis

In this section, we calibrate the model to illustrate quantitatively the growth and welfare effects of strengthening blocking patents (i.e., increasing s). First, we evaluate the effects of increasing s from 0 to 1 on steady-state welfare. Then, we simulate transition dynamics to compute complete welfare changes. Specifically, we consider two types of policy reform: (a) an immediate increase in s, and (b) a gradual increase in s.

5.1. Steady-state welfare

For the structural parameters, we either consider conventional parameter values or calibrate their values by using empirical moments in the US before the patent-policy reform in 1982. For the discount rate ρ, we set it to 0.03.[33] For the R&D externality parameters ϕ_q and ϕ_n, we consider the symmetric case of $\phi = \phi_q = \phi_n$ and follow Jones and Williams (2000) to consider a value of $\phi = 0.5$.[34] Similarly, we consider the symmetric case of $\varphi = \varphi_q = \varphi_n$ for R&D productivity as in Gersbach et al. (2009).[35] To calibrate the values of the remaining structural parameters φ, δ, z and L, we use the following four empirical moments: (i) the arrival rate of vertical innovation, (ii) the average growth rate of total factor productivity, (iii) R&D as a share of GDP, and (iv) the ratio of R&D scientists and engineers to manufacturing labor force. For (i), we follow Acemoglu and Akcigit (2012) to consider an innovation-arrival rate of $\lambda = 0.33$. For (ii), we consider a value of $g = 1.5\%$. For (iii), we use a value of $R\&D/GDP = w_h/(w_h + w_l L + n^*\pi) = 1.5\%$. For (iv), there were 711.8 thousands full-time equivalent R&D scientists and engineers in the US in 1982,[36] and there were 17.36 millions people in the US manufacturing in 1982.[37] Given these empirical moments, we have the following calibrated values $\{\varphi,\delta,z,L\} = \{0.64,0.12,1.02,24.38\}$.

Table 1 shows that an increase in s would stifle vertical innovation by decreasing the arrival rate of productivity improvements. Despite the increase in horizontal innovation, the overall growth rate eventually decreases. This finding is consistent with the recent concerns about patent protection stifling the innovation process. However, Table 1 also suggests an interesting possibility that despite the lower growth rate, steady-state welfare U in (35) increases due to the higher rate of horizontal innovation.[38,39] In this simulation, we find that steady-state welfare is maximized as $s \to 1$, and this result is

[33] We have also considered a higher discount rate of 0.05 and found that the qualitative implication of our results remains unchanged.

[34] While Kortum's (1992) estimated value for a parameter similar to ϕ is 0.2, Jones and Williams (2000) use the empirical estimates of the social return to R&D to show that a lower bound for ϕ is 0.5. Therefore, we use $\phi = 0.5$ as our benchmark.

[35] In this calibration exercise, we consider the benchmark case of symmetric R&D parameters because a more detailed calibration requires disaggregate data on vertical and horizontal R&D. Unfortunately, we do not know of such data. However, if we follow the interpretation of Aghion and Howitt (1996) to treat horizontal R&D mainly as basic research and vertical R&D as applied research, then we can consider the data on basic R&D as a benchmark. According to OECD: Main Science and Technology Indicators, basic R&D is about 0.33% of US GDP in 1982. In our model's calibration, about 26% of high-skill labor is allocated to horizontal R&D implying that horizontal R&D as a share of GDP is about 0.39%. Therefore, the calibration based on symmetric R&D parameters is roughly in line with the data.

[36] This data is obtained from National Science Foundation. See the number of full-time equivalent R&D scientists and engineers in the US.

[37] Data from the Bureau of Labour Statistics. It is useful to note that if we use the total labor force (instead of the manufacturing labor force), the calibrated value of L would be even larger implying even larger welfare gains.

[38] It is useful to note that this finding of a welfare gain is robust to the normalization of $q_0(i) = 0$ for all i. In the case of $q_0(i) = q > 0$ for all i, the welfare gain would have been more substantial because $q > 0$ has the same effect as a larger L as discussed before.

[39] We have also considered a hypothetical value of $s = 1.1$ and find that welfare continues to increase in s. This result also applies to the subsequent results with transition dynamics. However, a potential problem with $s > 1$ is that if patent infringement occurs only when an entrant launches her

Table 1
Effects of s on growth and welfare.

s	0.0	0.2	0.4	0.6	0.8	1.0
λ	0.33	0.30	0.27	0.25	0.22	0.18
g	1.500%	1.513%	1.505%	1.474%	1.413%	1.301%
U	217.7	228.9	238.9	247.9	256.2	264.0

driven by a relatively large value of $L=24.38$ (recall that L is the number of production workers per each R&D worker). Holding other parameter values constant, we find that in the case of decreasing L to about 12, the welfare-maximizing s_u becomes an interior solution.[40] If we further decrease L to about 7, steady-state welfare would be maximized as $s \to 0$. In summary, these illustrative numerical exercises suggest the importance of taking into consideration the stimulating effect of s on horizontal innovation in order to perform a proper welfare analysis.

5.2. Immediate patent reform

In the previous section, we evaluated the effects of an increase in s on steady-state welfare. However, such an analysis neglects the welfare changes during the transition path. Therefore, in this section, we simulate transition dynamics of the model.[41] Given the transition path of the consumption index, we can then evaluate the complete welfare effects of an immediate increase in s from $s=0$ to $s \in \{0.2,0.4,0.6,0.8,1.0\}$. Comparing Tables 1 and 2, we see that increasing s would improve welfare even taking into consideration transition dynamics. However, the magnitude of the welfare improvement is smaller than in the case of steady-state welfare.

5.3. Gradual patent reform

In the previous section, we evaluated the welfare effects of an immediate increase in s. However, in the US, the patent reform may be more accurately described as a gradual reform. For example, in 1982, the US Congress established the Court of Appeals for the Federal Circuit (CAFC) as a centralized appellate court for patent cases. "Over the next decade, in case after case, the court significantly broadened and strengthened the rights of patent holders."[42] Also, the Ginarte–Park index (described in Section 1) shows that the strength of patent protection in the US *gradually* increases from 3.8 in 1975 to 4.9 in 1995 (Table 3).[43]

Therefore, in this section, we evaluate the welfare effects of a gradual increase in s from $s=0$ to $\bar{s} \in \{0.2,0.4,0.6,0.8,1.0\}$. Following Cozzi and Galli (2011), we consider a law of motion for s_t given by

$$\dot{s}_t = \psi(\bar{s} - s_t), \tag{40}$$

where the parameter $\psi \in (0,1)$ determines the speed of the patent reform. In the numerical exercise, we consider $\psi = 0.05$ for illustrative purposes. Table 4 shows that a gradual increase in s would improve social welfare but by a smaller magnitude than an immediate increase in s. Furthermore, the welfare gain is increasing in ψ (i.e., increasing in the speed of reform). As ψ approaches one, the welfare gain becomes the same as in Section 5.2.

6. Extensions

In this section, we consider two important extensions of the previous setting. In the first extension, we analyze a different institutional setting that allows stronger patents to discriminate in favor of horizontal innovation only. In other words, under the basic-research interpretation of horizontal innovation, in Section 6.1, we consider an alternative profit-division rule under which only the basic researcher can appropriate a fraction of the profits created by all future applied innovations in the industry.

In Section 6.2, we instead remove the assumption that skilled labor is segregated into the R&D sectors while unskilled labor is employed only in the manufacturing sector. In this section, we will consider the case of homogeneous labor employable in all sectors. As we shall see, both variants of the basic analysis lead to similar conclusions, though they

(*footnote continued*)
product in the market (rather than when she comes up with the innovation), she may not have the incentives to launch her high-quality product to avoid paying the penalty to the incumbent. If every subsequent entrant acts in this way, then vertical innovation would come to a halt.

[40] Interestingly, there were 1.1 million full-time equivalent R&D scientists and engineers in 2007, according to NSF. Since the manufacturing labour force was 13.8 millions, the resulting $L=11.54$, which implies an optimal level of s around 0.8.

[41] See Appendix B for a description of the dynamic system and the numerical algorithm.

[42] Jaffe and Lerner (2004, p. 9–10).

[43] The Ginarte–Park index is an aggregate measure of patent rights rather than a direct measure of the profit-division rule. Although an empirical measure of "s" is not available, the anecdotal evidence from Jaffe and Lerner (2004) seems to suggest that it increases gradually in the US rather than once and for all in the early 1980s.

Table 2
Welfare effects of an immediate increase in s.

s	0.0	0.2	0.4	0.6	0.8	1.0
U (*transition*)	217.7	226.8	235.1	242.5	249.2	255.3

Table 3
Index of patent rights from Park (2008).

Year	1975	1980	1985	1990	1995	2000	2005
United States	3.83	4.35	4.68	4.68	4.88	4.88	4.88

Table 4
Welfare effects of a gradual increase in s.

s	0.0	0.2	0.4	0.6	0.8	1.0
U ($\psi=0.05$)	217.7	224.3	230.4	236.0	240.3	245.8

weaken the calibrated optimal backloading parameter s to a value slightly less than 1, which nevertheless is still substantially larger than the growth-maximizing share.

6.1. An alternative profit-division rule

The results of the numerical analysis of the previous section shows that perfect backloading maximizes welfare. We share the view[44] that this depends on our assumption that the inventors of basic technologies cannot obtain a share of the profit following a second improvement in the technology. Within this quite restrictive institutional setting, even the strongest protection of basic technologies, that is, $s=1$, allows the basic innovator in each industry to appropriate only part of the total profit flow generated by her basic R&D result. If we instead assume that basic inventors obtain a share s of the industry's profit, i.e., that in each industry profit gets divided between the inventor of the basic technology and the inventor of the current state-of-the-art quality, our numerical results show that the perfect protection of basic research patents, that is, $s=1$, would not maximize welfare any more.

Let us indeed assume that the basic researcher is entitled to a share s of the profit of all future applied researchers using her innovation, that is, a share of the profits of all the future firms in the industry until the industry becomes obsolete. We focus on the steady-state analysis. It can be noticed that, under the new framework, all the previous equations continue to hold, with the exception of Eqs. (11), (12), (16), (23), (24) and (25).

Let us begin by denoting $v_{2,t}(i)$ as the value of the patent held by the second-most recent innovator in industry i, provided she was *not* the first innovator in the industry: that is, who was an applied researcher, not a basic researcher. Then, (11) now becomes $v_{2,t}=0$, because a former incumbent is not entitled to any share of the profits of successive innovators. Consequently, the previous condition in (12) is modified to

$$r_t v_{1,t} = (1-s)\pi_t + \dot{v}_{1,t} - (\delta+\lambda_t)v_{1,t}, \tag{41}$$

because, when the next quality improvement occurs, the most recent applied R&D innovator is driven out of the market.

The horizontal innovator's arbitrage Eq. (16) is now

$$r_t v_{n,t} = \pi_t + \dot{v}_{n,t} - (\delta+\lambda_t)v_{n,t} + \lambda_t v_{B,t}, \tag{42}$$

where $v_{B,t}$ is the expected discounted value of the stream of royalties $s\pi_{\tau \geq t}$ from all future applied innovators in the industry. The no-arbitrage condition for $v_{B,t}$ is

$$r_t v_{B,t} = s\pi_t + \dot{v}_{B,t} - \delta v_{B,t}. \tag{43}$$

Intuitively, (43) equates the interest rate to the per unit asset return given by the sum of (a) the profit received by the horizontal innovator $s\pi_t$, (b) any potential capital gain $\dot{v}_{B,t}$, and (c) the expected capital loss due to obsolescence $\delta v_{B,t}$ only (i.e., creative destruction no longer affects $v_{B,t}$).

As in the analysis of Section 4, in a stationary equilibrium, ζ is constant, $r_t=\rho$, and $\dot{\pi}_t/\pi_t=0$. Therefore, we can write

$$v_{B,t} = \frac{s\pi}{\rho+\delta} \equiv v_B. \tag{44}$$

[44] We are indebted to a Referee for this very useful suggestion.

Table 5
Effects of s on growth and welfare.

s	0.0	0.2	0.4	0.6	0.8	1.0
λ	0.33	0.25	0.19	0.14	0.08	0
g	1.50%	1.49%	1.35%	1.10%	0.71%	0%
U	217.7	245.4	261.3	270.7	274.8	271.3

Applying all previous equations, we derive the new version of previous (23) as

$$v_1 = \frac{(1-s)\pi}{\rho+\delta+\lambda}, \tag{45}$$

and of previous (24) as

$$v_n = \frac{\pi+\lambda v_B}{\rho+\delta+\lambda} = \frac{\pi}{\rho+\delta+\lambda}\left(1+\frac{s\lambda}{\rho+\delta}\right). \tag{46}$$

Substituting these new versions of (23) and (24) into $v_1\bar{\varphi}_q = v_n\bar{\varphi}_n$ yields

$$(h_n)^{1-\phi_n} = \left(\frac{\varphi_n}{\varphi_q}\frac{\rho+\delta+s\varphi_q(h_q)^{\phi_q}}{(1-s)(\rho+\delta)}\right)(h_q)^{1-\phi_q}. \tag{47}$$

The resource constraint for skilled labor continues to be

$$\frac{\delta(1-h_n)}{\varphi_n(h_n)^{\phi_n}} = h_q. \tag{48}$$

Everything else remains unchanged.

Because the new arbitrage condition in (47) represents a positive relationship between h_n and h_q as before and the resource constraint remains unchanged, Fig. 1 also applies to this alternative profit-division rule. An increase in s rotates the new AC curve upwards while leaving the RC curve unaffected; consequently, we obtain the same result as before that the equilibrium $h_n(s)$ is increasing in s whereas $h_q(s)$ is decreasing in s. Furthermore, by comparing the old AC curve (25) and the new AC curve (47), one can see that the two AC curves are identical when $s=0$. Given that the RC curve is the same in both cases, the equilibrium allocation under the alternative profit-division rule is the same as in the baseline model when $s=0$. However, as s increases above zero, it can be shown that the new AC curve is positioned above the old AC curve in Fig. 1 implying that for each value of $s>0$, the equilibrium $h_n(s)$ is higher under the alternative profit division rule than under the baseline model. Given that the growth-maximizing $h_n(s_g)$ given by Φ in (34) is independent of the profit-division rule, the growth-maximizing s_g must be lower under the alternative profit-division rule than under the baseline model.

Regarding the quantitative analysis, we note that the calibrated parameters are the same as before, because their derivation is based on the common benchmark case in which $s=0$. Therefore we can use $\{\varphi,\delta,z,L\} = \{0.64, 0.12, 1.02, 24.38\}$. We have simulated this variant of the model and have found qualitative results that are similar to those of the previous sections. Most notably, a smaller L would be associated with a lower value of the welfare-maximizing s, as would a lower discount rate. In other words, an economy with relatively more educated workers (i.e., a smaller L) and/or more patient people (i.e., a smaller ρ) would value growth more and would be less avid for varieties. The following Table 5 summarizes our findings for different levels of s.

From these results we notice that, consistent with what conjectured, it is not convenient for the economy to suppress vertical innovation, by adopting a confiscatory rate on applied R&D. Moreover, by comparing the entries of Table 5 with those of Table 2, we can see that channeling applied R&D profits only to basic research, rather than generically backloading profits to previous innovators regardless of its research being basic or applied, as we have done in the previous sections, would increase the attainable levels of welfare. In fact, steady state welfare $U=274.8$ would not be attainable in the previous simulations. More precisely, by digging further into this 1982 US scenario, it can be shown that the steady-state welfare maximizing basic research share would be $s=0.84$ (with $U=274.9$). Finally, the growth-maximizing share is $s=0.08$, which is substantially smaller than the welfare-maximizing share.

6.2. Homogeneous labor

In the previous sections, we have focused on a model in which there are unskilled workers for production and skilled workers for vertical and horizontal R&D. This exogenous separation between production workers and R&D workers helps simplifying the analytical derivations. To examine the robustness of our main results, we consider the case of homogeneous workers for production, vertical and horizontal R&D in this section.[45]

[45] The authors would like to thank the Referees for this very helpful suggestion.

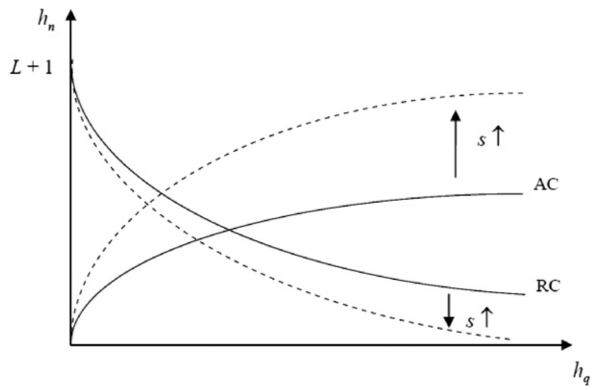

Fig. 3. Homogeneous labor.

Table 6
Effects of s under homogeneous labor.

s	0.0	0.2	0.4	0.6	0.8	1.0
λ	0.33	0.29	0.25	0.21	0.16	0.06
g	1.50%	1.42%	1.31%	1.16%	0.92%	0.40%
U	129.0	134.1	138.3	141.8	144.4	144.3

Under homogeneous labor, $w_{h,t}= w_{l,t}=w_t$, and the total supply of labor is $L+1$. The rest of the model is the same as in Section 2 except for the new resource constraint (RC) on labor as follows.

$$h_{n,t}+n_t^* h_{q,t}+n_t^* l_t = L+1. \tag{49}$$

In the stationary equilibrium, the arbitrage condition (AC) between vertical and horizontal R&D continues to be given by (25). To close the model, we equate (10) and (15) to obtain

$$\frac{\pi_t}{(z-1)l_t} = w_t = \bar{\varphi}_{q,t} v_{1,t}, \tag{50}$$

where we normalize w_t to unity. Substituting (23) and $\lambda=\varphi_q(h_q)^{\phi_q}$ into (50), we obtain

$$\frac{1}{l} = \frac{(z-1)\varphi_q(h_q)^{\phi_q-1}}{\rho+\delta+\varphi_q(h_q)^{\phi_q}}\left(1-s+s\frac{\varphi_q(h_q)^{\phi_q}}{\rho+\delta+\varphi_q(h_q)^{\phi_q}}\right). \tag{51}$$

Substituting (21) and (51) into (49) and then rearranging terms yield

$$\frac{\delta(L+1-h_n)}{\varphi_n(h_n)^{\phi_n}} = h_q+l = h_q+\left(\frac{(h_q)^{1-\phi_q}}{(z-1)\varphi_q}\right)\frac{[\rho+\delta+\varphi_q(h_q)^{\phi_q}]^2}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}. \tag{52}$$

It can be shown that $\phi_q \le 0.5$ is sufficient for the new RC in (52) to exhibit a monotonically negative relationship in the (h_q,h_n) space for all values of $s\in[0,1]$; therefore, solving the AC in (25) and the new RC in (52) yields the unique equilibrium allocation of h_n and h_q as before. An increase in s rotates the AC upwards as before, whereas it also rotates the new RC downwards as shown in Fig. 3. Therefore, the additional general-equilibrium effect through a reallocation of production workers strengthens the negative effect of s on vertical R&D h_q and weakens the positive effect of s on horizontal R&D h_n. It may seem that the net effect of a higher level of s on horizontal R&D h_n is ambiguous. However, we can prove the following result:

Lemma 3. *The steady-state level of horizontal R&D h_n is an increasing function of s.*

Proof. See Appendix A. □

The steady-state equilibrium growth rate and welfare continue to be given by (30) and (35) respectively; however, the equilibrium allocation of l is now given by (51). To examine how an increase in s affects economic growth and social welfare, we calibrate the parameters as before. First, we set the discount rate to $\rho=0.03$ and the R&D externality parameter to $\phi=\phi_q=\phi_n=0.5$. For the remaining parameters $\{\varphi,\delta,z,L\}$, we calibrate them using (i) $\lambda=0.33$, (ii) $g=1.5\%$, and (iii) the ratio of R&D workers to labor force given by $(h_n+n^*h_q)/(L+1)$. Finally, we set L=24.38 as in Section 5.1, so that the two calibrated economies have the same size of labor force. The calibrated parameter values are $\{\varphi,\delta,z\}=\{0.48,0.30,1.04\}$. Table 6 reports the results. The qualitative pattern of the results under homogeneous labor is consistent with the results under heterogeneous labor. As s increases, the arrival rate of vertical innovation decreases, whereas the equilibrium growth rate becomes monotonically decreasing in s providing further support for the innovation–stifling effect of blocking patents. Finally, social welfare becomes an inverted-U function in s with a welfare-maximizing value of 0.93 once again showing the different implications of growth maximization versus welfare maximization.

7. Conclusion

In this study, we have developed a simple growth model to shed some light on an often debated question that is whether patent protection stimulates or stifles innovation. We show that both sides of the argument are valid. Specifically, protecting incumbents at the expense of entrants would stimulate horizontal innovation but stifle vertical innovation, and the opposite occurs when entrants are protected against incumbents. Although the distinction between vertical and horizontal innovation is blurred in reality, our point is still valid in the sense that patent protection has asymmetric effects on different types of innovation that carry different chances of patent infringements, and hence, the traditional tradeoff of optimal patent protection needs to be modified to take into account these asymmetric effects of patent policy. In other words, optimal patent policy should be innovation-specific. If vertical (horizontal) innovation is crucial to social welfare, then a more frontloading (backloading) profit-division rule should be implemented. Furthermore, if we follow Aghion and Howitt (1996) to treat horizontal R&D as basic research and vertical R&D as applied research, then our finding implies that a gradual increase in the bargaining power of basic researchers could be welfare-improving, and this finding is consistent with the two-stage R&D analysis in Cozzi and Galli (2011), who consider a transition to more upstream bargaining power.

Finally, in this study, we have also considered an alternative profit-division rule such that the variety inventor of an industry always obtains a share s of the monopolistic profits generated by all subsequent innovations in the industry. This has helped test the robustness of the main economic effects we have found, and allowed us to discriminate more precisely regarding the optimality of different ways of strengthening patent protection. Overall, we hope that our simple model has served the purpose of highlighting the asymmetric effects of patent rights on different types of innovation and the potentially different policy implications on economic growth and social welfare.

Acknowledgments

The authors are grateful to the anonymous Referees and Theo Eicher (the Editor) for their insightful comments. We also would like to thank the seminar and conference participants at Academia Sinica, Chinese University of Hong Kong, Durham University, ETH Zurich, Kyoto University, National Taiwan University, Royal Economic Society, Shanghai Macroeconomic Workshop, and Taipei International Conference on Growth, Trade and Dynamics for their helpful suggestions. Chu is also grateful for the support by the Leading Academic Discipline Program, 211 Project for Shanghai University of Finance and Economics (the fourth phase). The usual disclaimer applies. The previous version of this paper was circulated under the title "Innovation-Specific Patent Protection".

Appendix A. Proofs

Proof of Lemma 1. From (23), the value of a quality improvement is $v_1 = \pi/(\rho+\delta+\lambda)(1-s+s\lambda/(\rho+\delta+\lambda))$ for a firm that does not own the previous innovation. For an incumbent (i.e., a firm that owns the previous innovation), the incremental value of a quality improvement is $v_I = \pi/(\rho+\delta+\lambda)(1+s\lambda/(\rho+\delta+\lambda))-v_2$.[46] The first term in v_I reflects that the firm's new product infringes its own patent and hence it does not have to pay any licensing fee. The second term (i.e., $-v_2$) reflects that the incumbent's old invention loses the opportunity to extract profit from the new entrant. Substituting $v_2 = s\pi/(\rho+\delta+\lambda)$ into v_I yields $v_I = v_1$ for $s \in [0,1]$, so that the incumbent is indifferent as to where to target innovation. As a result, all the aggregate variables behave as if quality improvement is targeted only by the entrants (i.e., the Arrow replacement effect).[47] □

Proof of Lemma 2. Lets firstly define a new variable $x \equiv \varphi_q(h_q)^{\phi_q}$ and a new function

$$f(x) \equiv \frac{1}{\rho+\delta+(1+s)x}\left(\frac{x}{(1-s)(\rho+\delta)+x}\right). \tag{A.1}$$

[46] To be consistent with the assumption of no market-power consolidation, an upper bound of z is imposed on the markup, so that π is the same in v_1 and v_I. In the case of market-power consolidation, the markup would be given by z^2 regardless of whether or not the two generations of quality improvement are owned by the same firm, so that π would be the same in v_1 and v_I as well.

[47] This new interpretation of the Arrow effect is developed by Cozzi (2007), who shows that the incumbent's current invention faces the same probability of being displaced regardless of whether or not an incumbent targets innovation at her own industry. Under the traditional interpretation (i.e., when an incumbent obtains a new invention, she loses the value of the old invention), it should be v_1 (instead of v_2) that is substracted from v_I. In this case, $v_I = \pi/(\rho+\delta+\lambda)(1+s\lambda/(\rho+\delta+\lambda))-v_1 = s\pi/(\rho+\delta+\lambda)$, and hence $v_I < v_1 \Longleftrightarrow s < \hat{s} \equiv (\rho+\delta+\lambda)/(2(\rho+\delta)+\lambda) \in [0.5,1]$. Therefore, when $s < \hat{s}$, quality improvement is targeted by entrants only, so that the Arrow replacement effect is again present.

Simple differentiation yields

$$\arg\max f(x) = (\rho+\delta)\sqrt{\frac{1-s}{1+s}}. \tag{A.2}$$

Given that dh_n/dh_q in (28) is decreasing in $f(x)$, maximizing $f(x)$ is equivalent to minimizing the bracketed term in (28). Substituting (A.2) into (28) yields

$$\frac{dh_n}{dh_q} = \frac{1}{1-\phi_n}\left(1-\phi_q-\phi_q\frac{s^2}{2-s^2+2\sqrt{1-s^2}}\right)\frac{h_n}{h_q}. \tag{A.3}$$

Manipulating (A.3) shows that $\phi_q < [1-0.5s^2/(1+\sqrt{1-s^2})] \in [0.5,1]$ implies $dh_n/dh_q > 0$ in (28) for any value of $h_q > 0$. □

Proof of Lemma 3. In what follows, we show that h_n is always increasing in s under homogeneous labour. Recall from the resource constraint (52) that

$$\frac{\delta(L+1-h_n)}{\varphi_n(h_n)^{\phi_n}} = h_q + l.$$

We already know that h_q is decreasing in s. Therefore, if we can show that l is also decreasing in s, then h_n must be increasing in s. Also, it is useful to note that $\partial l/ds > 0$ is a *necessary* condition for $\partial h_n/ds < 0$. Rewriting (51), we have

$$l = \frac{(h_q)^{1-\phi_q}}{(z-1)\varphi_q}\frac{[\rho+\delta+\varphi_q(h_q)^{\phi_q}]^2}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}. \tag{51A}$$

For the special case of $\rho+\delta\to 0$, it is easy to see that $\partial l/\partial s$ must have the same sign as $\partial h_q/\partial s < 0$. As for the general case of $\rho+\delta > 0$, differentiating the log of (51A), we have

$$\frac{1}{l}\frac{\partial l}{\partial s} = \underbrace{\left(\frac{1-\phi_q}{h_q}+\frac{2\varphi_q\phi_q(h_q)^{\phi_q-1}}{\rho+\delta+\varphi_q(h_q)^{\phi_q}}-\frac{\varphi_q\phi_q(h_q)^{\phi_q-1}}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}\right)}_{\equiv A}\underbrace{\frac{\partial h_q}{\partial s}}_{<0}+\frac{(\rho+\delta)}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}$$

It is useful to note that the parameter condition $\phi_q \le 0.5$ that we assume throughout the analysis is sufficient for the term A to be positive. To see this, A can be expressed as

$$\begin{aligned} A &= \frac{1}{h_q}\left[1-\phi_q+\varphi_q\phi_q(h_q)^{\phi_q}\left(\frac{(1-2s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}{[\rho+\delta+\varphi_q(h_q)^{\phi_q}][(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}]}\right)\right] \\ &= \frac{1}{h_q}\left[\frac{[2-(s+\phi_q+s\phi_q)](\rho+\delta)\varphi_q(h_q)^{\phi_q}+(\rho+\delta)^2(1-\phi_q)(1-s)+(2-\phi_q)[\varphi_q(h_q)^{\phi_q}]^2}{[\rho+\delta+\varphi_q(h_q)^{\phi_q}][(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}]}\right] \end{aligned}$$

It is now straightforward to see that the only term in A that can be negative is $2-(s+\phi_q+s\phi_q)$, which is positive given $\phi_q \le 0.5$. Given $A>0$, it must be the case that $\partial l/\partial s < 0$ if and only if the following inequality holds: $\partial h_q/\partial s < -[(\rho+\delta)/((1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q})]/A$.

From the arbitrage condition in (25), we have

$$(h_n)^{1-\phi_n} = \left(\frac{\varphi_n}{\varphi_q}\frac{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}\right)(h_q)^{1-\phi_q}. \tag{25A}$$

Taking the log of (25A) and then differentiating with respect to s yields

$$\begin{aligned} \frac{1-\phi_n}{h_n}\frac{\partial h_n}{\partial s} &= \frac{(\rho+\delta)}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}+\frac{\varphi_q(h_q)^{\phi_q}}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}} \\ &\quad+\underbrace{\left[\frac{1-\phi_q}{h_q}-\frac{\varphi_q\phi_q(h_q)^{\phi_q-1}}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}}\frac{(\rho+\delta)s^2}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}\right]}_{\equiv B}\frac{\partial h_q}{\partial s}. \end{aligned}$$

If $B<0$, then $\partial h_n/\partial s$ must be positive because $\partial h_q/\partial s < 0$. So, we only have to further analyze the case in which $B>0$, under which the sign of $\partial h_n/\partial s$ appears to be ambiguous. Here we consider a proof by contradiction. Suppose $\partial h_n/\partial s < 0$. Then, the following inequality must hold:

$$\frac{\partial h_q}{\partial s} < -\left[\frac{(\rho+\delta)}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}+\frac{\varphi_q(h_q)^{\phi_q}}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}}\right]\frac{1}{B} \tag{A.4}$$

However, we will show that if this inequality holds, then $\partial l/\partial s$ would be negative, contradicting our initial assumption $\partial h_n/\partial s<0$, which requires $\partial l/\partial s>0$.

In the remaining analysis, we show that

$$-\left[\frac{(\rho+\delta)}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}+\frac{\varphi_q(h_q)^{\phi_q}}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}}\right]\frac{1}{B}<-\left[\frac{(\rho+\delta)}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}\right]\frac{1}{A}. \tag{A.5}$$

Notice that (51A), by (A.4), implies $\partial h_q/\partial s<-[(\rho+\delta)/((1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q})]/A$, and therefore $\partial l/\partial s<0$. Inequality (51A) can be re-expressed as

$$\left[1+\frac{(1-s)\varphi_q(h_q)^{\phi_q}+[\varphi_q(h_q)^{\phi_q}]^2/(\rho+\delta)}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}}\right]A>B.$$

Therefore, it suffices to show that $A\geq B$, which can written as

$$1+\frac{s^2(\rho+\delta)+(1+s^2)\varphi_q(h_q)^{\phi_q}}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}}+\frac{(1+s)[\varphi_q(h_q)^{\phi_q}]^2/(\rho+\delta)}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}}\geq 2s,$$

which holds if

$$1+\frac{s^2(\rho+\delta)+(1+s^2)\varphi_q(h_q)^{\phi_q}}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}}\geq 2s.$$

Since $(s^2(\rho+\delta)+(1+s^2)\varphi_q(h_q)^{\phi_q})/(\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q})$ is decreasing in $(\rho+\delta)$, it suffices to show—after letting $(\rho+\delta)$ tend to infinity—that

$$1+s^2\geq 2s \Leftrightarrow 1\geq s(2-s),$$

which holds because $s(2-s)$ reaches its maximum at $s=1$. □

Appendix B. Transition dynamics

The system of equations that characterizes the dynamics of the model is as follows:

$$\dot{n}_t^*=\varphi_n(h_{n,t})^{\phi_n}-\delta n_t^*, \tag{B.1}$$

$$\dot{\zeta}_t/\zeta_t=\rho-r_t, \tag{B.2}$$

$$\dot{\nu}_{2,t}=(r_t+\lambda_t+\delta)\nu_{2,t}-s\pi_t, \tag{B.3}$$

$$\dot{\nu}_{1,t}=(r_t+\lambda_t+\delta)\nu_{1,t}-\lambda_t\nu_{2,t}-(1-s)\pi_t, \tag{B.4}$$

$$\dot{\nu}_{n,t}=(r_t+\lambda_t+\delta)\nu_{n,t}-\lambda_t\nu_{2,t}-\pi_t, \tag{B.5}$$

$$\pi_t=\left(\frac{z-1}{z}\right)\frac{1}{\zeta_t}, \tag{B.6}$$

$$\lambda_t=\varphi_q(h_{q,t})^{\phi_q}, \tag{B.7}$$

$$\nu_{1,t}\varphi_q(h_{q,t})^{\phi_q-1}=\nu_{n,t}\varphi_n(h_{n,t})^{\phi_n-1}, \tag{B.8}$$

$$h_{n,t}+n_t^*h_{q,t}=1, \tag{B.9}$$

$$n_t^*l_t=L, \tag{B.10}$$

$$\pi_t=(z-1)w_{l,t}l_t=\left(\frac{z-1}{z}\right)\frac{1}{\zeta_t}\Longrightarrow zw_{l,t}l_t=\frac{1}{\zeta_t}. \tag{B.11}$$

Finally, we choose l_t as the numeraire by setting $w_{l,t}=1$. The endogenous variables in this system are $\{n_t^*,\zeta_t,\nu_{2,t},\nu_{1,t},\nu_{n,t},\pi_t,\lambda_t,\ h_{q,t},h_{n,t},l_t,r_t\}$.

In all our numerical simulations, in order to simulate the dynamic transition from one steady state to another, we first compute the initial steady state and the final steady state, associated with the initial and final level of s; then we discretize all the differential equations in system (B.1)–(B.11), and plug them as well as the remaining equation restrictions in a .mod file, which allows Dynare to apply its deterministic routines, needed to compute the dynamic rational expectations equilibrium transition from the initial to the final steady state. Since Dynare also analyses the eigenvalues of the Jacobian matrix at the final steady state, while simulating the transitional path we always make sure that in all our simulations the conditions for the determinacy of the steady state are satisfied, that is the number of stable eigenvalues is equal to the number of predetermined variables. Hence, all the transitional paths we have obtained are along the unique equilibrium of the economy analyzed.

In order to calculate the complete change in welfare, we need to keep track of the evolution of the consumption index.

$$\ln c_t = \int_0^{n_t^*} (q_t(i)\ln z + \ln l_t(i))\, di = \left(\int_0^{n_t^*} q_t(i)\, di\right)\ln z + n_t^* \ln l_t. \tag{B.12}$$

Normalizing $q_0(i) = 0$ for all i, we can re-express the level of aggregate technology as

$$\int_0^{n_t^*} q_t(i)\, di = \int_0^t n_\tau^* \lambda_\tau\, d\tau + \int_0^t \dot{n}_\tau^* \left(\int_0^\tau \lambda_\upsilon\, d\upsilon\right) d\tau. \tag{B.13}$$

The first term on the right hand side of (B.13) is the accumulated number of productivity improvements that have occurred from time 0 to time t. The second term on the right hand side of (B.13) is the change in aggregate technology due to the introduction of new varieties net of obsolescence. Using the data generated by Dynare, we could then compute the discretized version of the welfare integral, which allowed the welfare experiments reported in the tables of Section 5.

Notice that by normalizing $q_0(i) = 0$ for all i, in light of (B.13), we are minimizing the effect of $\dot{n}_t^*$ on welfare. This proves the robustness of the welfare comparisons in Tables 2 and 4. Given that n_t^* increases from the initial steady state to the new steady state in our numerical exercises, any alternative positive level of the $q_0(i)$'s would imply a higher transitional welfare effect of an increase in s.

References

Acemoglu, D., 2009. A note on diversity and technological progress, manuscript.

Acemoglu, D., Akcigit, U., 2012. Intellectual property rights policy, competition and innovation. Journal of the European Economic Association 10, 1–42.

Acs, Z., Sanders, M. Patents, knowledge spillovers, and entrepreneurship. Small Business Economics, doi:10.1007/s11187-011-9322-y, in press.

Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. Econometrica 60, 323–351.

Aghion, P., Howitt, P., 1996. Research and development in the growth process. Journal of Economic Growth 1, 49–73.

Bessen, J., Meurer, M., 2008. Patent Failure: How Judges, Bureaucrats, and Lawyers Put Innovators at Risk. Princeton University Press, Princeton, NJ.

Boldrin, M., Levine, D., 2008. Against Intellectual Monopoly. Cambridge University Press, New York, NY.

Boldrin, M., Levine, D., Nuvolari, A., 2008. Do patents encourage or hinder innovation? The case of the steam engine. The Freeman: Ideas on Liberty 58 (10).

Chu, A., 2009. Effects of blocking patents on R&D: a quantitative DGE analysis. Journal of Economic Growth 14, 55–78.

Chu, A., Pan, S. The escape-infringement effect of blocking patents on innovation and economic growth. Macroeconomic Dynamics, doi:10.1017/S136510051100068X, in press.

Cozzi, G., 2001. Inventing or spying? Implications for growth. Journal of Economic Growth 6, 55–77.

Cozzi, G., 2005. Animal spirits and the composition of innovation. European Economic Review 49, 627–637.

Cozzi, G., 2007. The Arrow effect under competitive R&D. B.E. Journal of Macroeconomics, 7 (1) (Contributions, Article 2).

Cozzi, G., Galli, S., 2011. Upstream Innovation Protection: Common Law Evolution and the Dynamics of Wage Inequality. MPRA Working Papers No. 31902.

Cozzi, G., Giordani, P., Zamparelli, L., 2007. The refoundation of the symmetric equilibrium in Schumpeterian growth models. Journal of Economic Theory 136, 788–797.

Cozzi, G., Spinesi, L., 2006. Intellectual appropriability, product differentiation, and growth. Macroeconomic Dynamics 10, 39–55.

Dinopoulos, E., Thompson, P., 1999a. Schumpeterian growth without scale effects. Journal of Economic Growth 3, 313–335.

Dinopoulos, E., Thompson, P., 1999b. Scale effects in Schumpeterian models of economic growth. Journal of Evolutionary Economics 9, 157–185.

Eicher, T., Garcia-Peñalosa, C., 2008. Endogenous strength of intellectual property rights: implications for economic development and growth. European Economic Review 52, 237–258.

Eswaran, M., Gallini, N., 1996. Patent policy and the direction of technological change. RAND Journal of Economics 27, 722–746.

Etro, F., 2004. Innovation by leaders. Economic Journal 114, 281–303.

Etro, F., 2008. Stackelberg competition with endogenous entry. Economic Journal 118, 1670–1697.

Furukawa, Y., 2007. The protection of intellectual property rights and endogenous growth: is stronger always better? Journal of Economic Dynamics and Control 31, 3644–3670

Futagami, K., Iwaisako, T., 2007. Dynamic analysis of patent policy in an endogenous growth model. Journal of Economic Theory 132, 306–334.

Gallini, N., 2002. The economics of patents: lessons from recent U.S. patent reform. Journal of Economic Perspectives 16, 131–154.

Gallini, N., Scotchmer, S., 2002. Intellectual property: when is it the best incentive system? In: Jaffe, A., Lerner, J., Stern, S. (Eds.), Innovation Policy and the Economy, vol. 2. , The MIT Press, Cambridge, MA, pp. 51–78.

Gersbach, H., Sorger, G., Amon, C., 2009. Hierarchical Growth: Basic and Applied Research. CER-ETH Economics Working Paper No. 09/118.

Ginarte, J., Park, W., 1997. Determinants of patent rights: a cross-national study. Research Policy 26, 283–301.

Green, J., Scotchmer, S., 1995. On the division of profit in sequential innovations. RAND Journal of Economics 26, 20–33.

Grossman, G., Helpman, E., 1991. Quality ladders in the theory of growth. Review of Economic Studies 58, 43–61.

Grossman, G., Lai, E., 2004. International protection of intellectual property. American Economic Review 94, 1635–1653.

Hall, B., Jaffe, A., Trajtenberg, M., 2001. The NBER Patent Citations Data File: Lessons, Insights and Methodological Issues. NBER Working Papers No. 8498.

Horii, R., Iwaisako, T., 2007. Economic growth with imperfect protection of intellectual property rights. Journal of Economics 90, 45–85.

Horowitz, A., Lai, E., 1996. Patent length and the rate of innovation. International Economic Review 37, 785–801.

Howitt, P., 1999. Steady endogenous growth with population and R&D inputs growing. Journal of Political Economy 107, 715–730.

Iwaisako, T., Futagami, K., 2003. Patent policy in an endogenous growth model. Journal of Economics 78, 239–258.

Iwaisako, T., Futagami, K. Patent protection, capital accumulation, and economic growth. Economic Theory, doi:10.1007/s00199-011-0658-y, in press.

Jaffe, A., 2000. The U.S. patent system in transition: policy innovation and the innovation process. Research Policy 29, 531–557.

Jaffe, A., Lerner, J., 2004. Innovation and Its Discontents: How Our Broken System is Endangering Innovation and Progress, and What to Do About It. Princeton University Press, Princeton, NJ.

Jones, C., 1999. Growth: with or without scale effects. American Economic Review 89, 139–144.

Jones, C., Williams, J., 2000. Too much of a good thing? The economics of investment in R&D. Journal of Economic Growth 5, 65–85.

Judd, K., 1985. On the performance of patents. Econometrica 53, 567–586.

Koleda, G., 2004. Patents' novelty requirement and endogenous growth. Revue d'economie politique 114, 201–221.

Kortum, S., 1992. Inventions, R&D and Industry Growth. Ph.D. Dissertation. Yale University.
Kortum, S., 1993. Equilibrium R&D and the patent-R&D ratio: U.S. evidence. American Economic Review 83, 450–457.
Li, C.-W., 2000. Endogenous vs. semi-endogenous growth in a two-R&D-sector model. Economic Journal 110, C109–C122.
Li, C.-W., 2001. On the policy implications of endogenous technological progress. Economic Journal 111, C164–C179.
Moser, P., 2005. How do patent laws influence innovation? Evidence from nineteenth-century World's Fairs. American Economic Review 95, 1214–1236.
Nordhaus, W., 1969. Invention, Growth, and Welfare: Theoretical Treatment of Technological Change. The MIT Press, Cambridge, MA.
O'Donoghue, T., Zweimuller, J., 2004. Patents in a model of endogenous growth. Journal of Economic Growth 9, 81–123.
Park, W., 2008. International patent protection: 1960–2005. Research Policy 37, 761–766.
Peretto, P., 1998. Technological change and population growth. Journal of Economic Growth 3, 283–311.
Peretto, P., 1999. Cost reduction, entry, and the interdependence of market structure and economic growth. Journal of Monetary Economics 43, 173–195.
Peretto, P., Smulders, S., 2002. Technological distance, growth and scale effects. Economic Journal 112, 603–624.
Scotchmer, S., 2004. Innovation and Incentives. The MIT Press, Cambridge, MA.
Segal, I., Whinston, M., 2007. Antitrust in innovative industries. American Economic Review 97, 1703–1730.
Segerstrom, P., 2000. The long-run growth effects of R&D subsidies. Journal of Economic Growth 5, 277–305.
Segerstrom, P., Anant, T., Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. American Economic Review 80, 1077–1092.
Shapiro, C., 2001. Navigating the patent thicket: cross licenses, patent pools, and standard setting. In: Jaffe, A., Lerner, J., Stern, S. (Eds.), Innovation Policy and the Economy, vol. 1. , The MIT Press, Cambridge, MA, pp. 119–150.
Thompson, P., 1996. Technological opportunity and the growth of knowledge: a Schumpeterian approach to measurement. Journal of Evolutionary Economics 6, 77–97.
Young, A., 1998. Growth without scale effects. Journal of Political Economy 106, 41–63.